

Solution to Problem 12)

$$\begin{aligned}\exp(A) \exp(B) &= (\sum_{m=0}^{\infty} A^m / m!) (\sum_{n=0}^{\infty} B^n / n!) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A^m B^n / (m! n!) \\ &= \sum_{k=0}^{\infty} \sum_{n=0}^k A^{k-n} B^n / [n! (k-n)!] = \sum_{k=0}^{\infty} \left[\sum_{n=0}^k \binom{k}{n} A^{k-n} B^n \right] / k! \\ &= \sum_{k=0}^{\infty} [(A+B)^k] / k! = \exp(A+B).\end{aligned}$$
